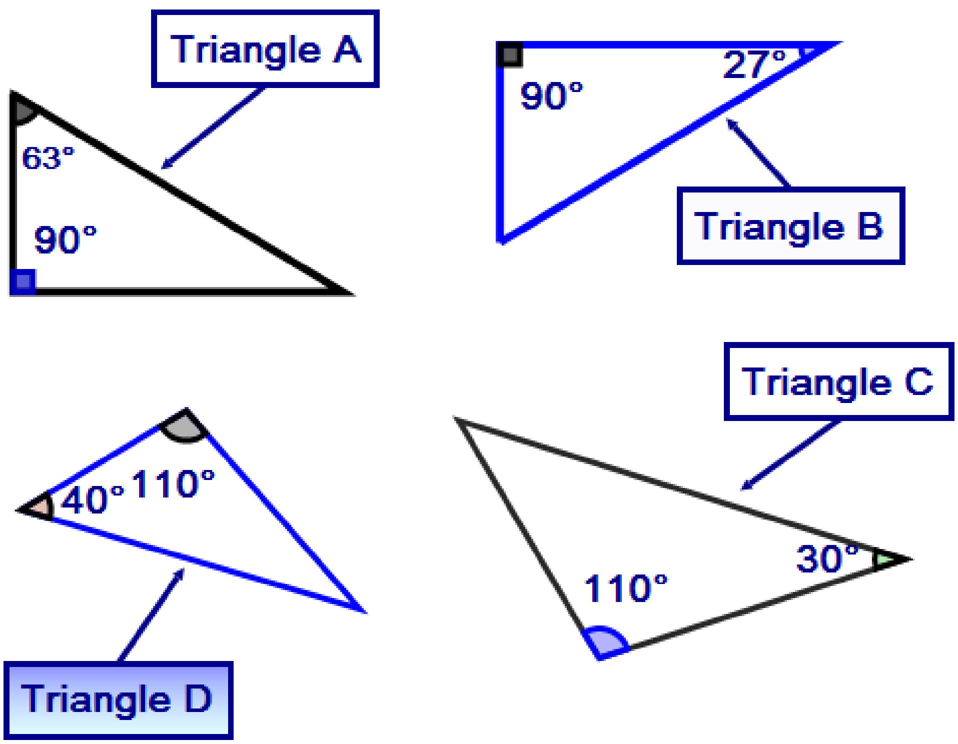
SIMILAR TRIANGLES/SHAPES. KS3 KS4. Non-Calculator

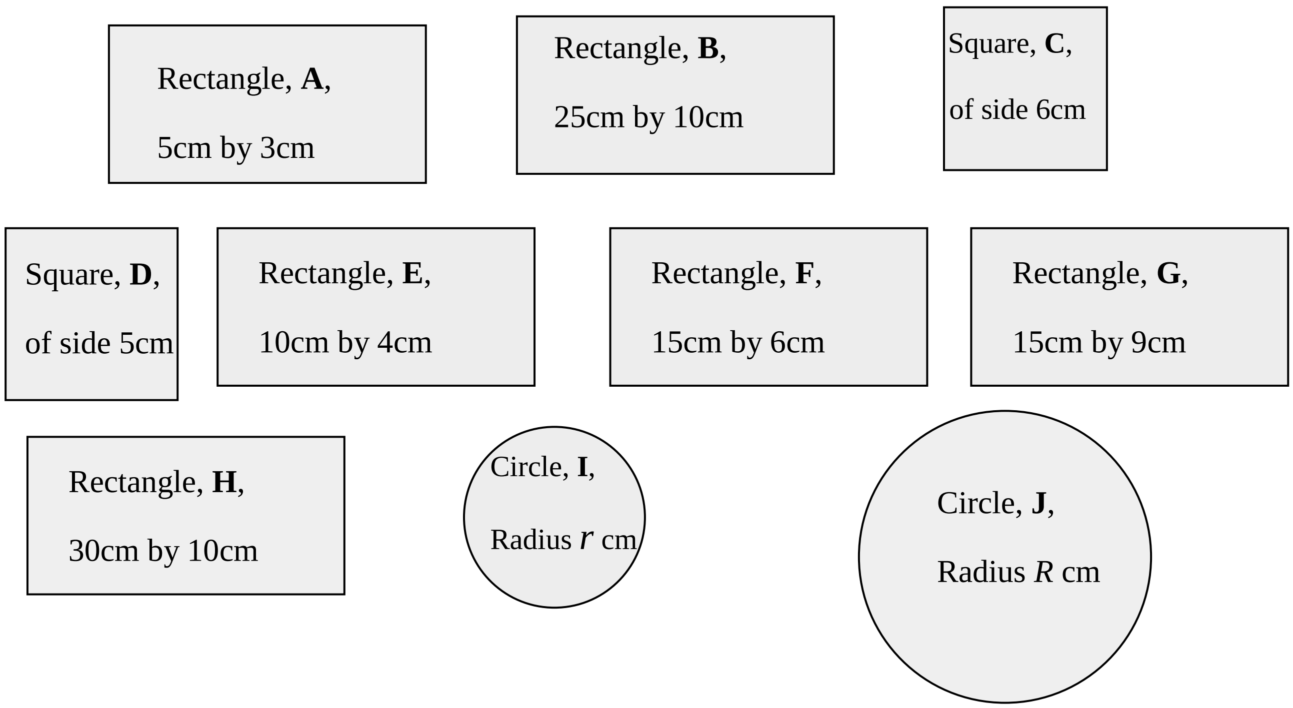
NOTE: ALL DIAGRAMS NOT DRAWN TO SCALE.

* Questions may be challenging for KS3.

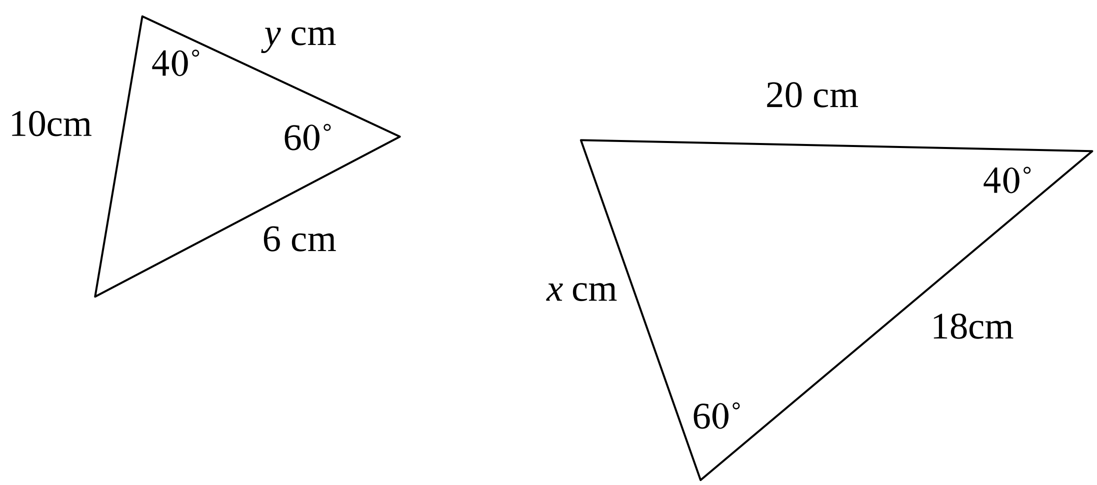
1. Which triangles are similar? Give reasons.



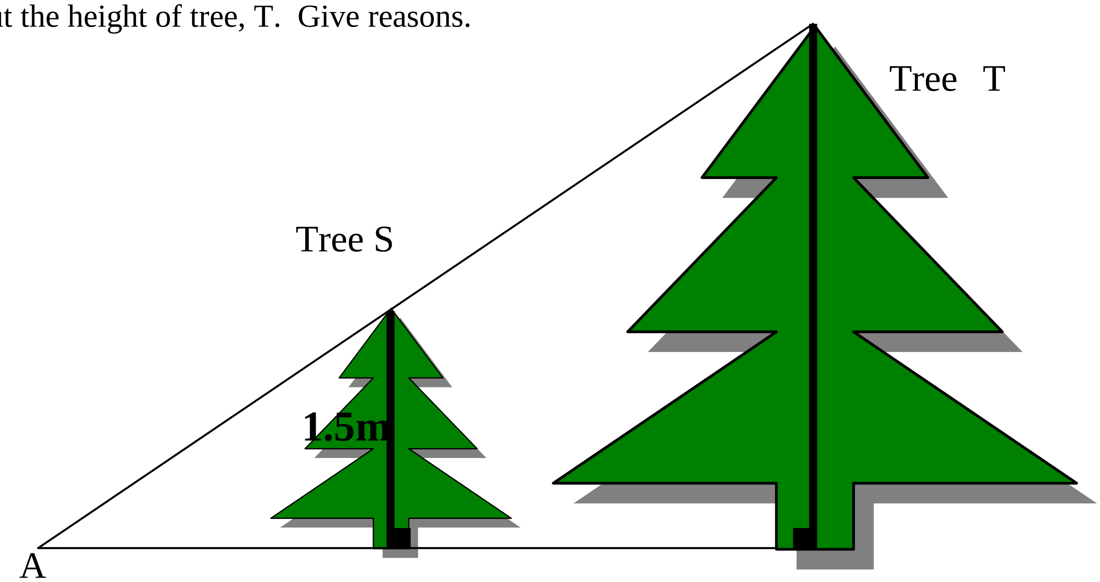
2. Which shapes are similar?
   Give reasons.



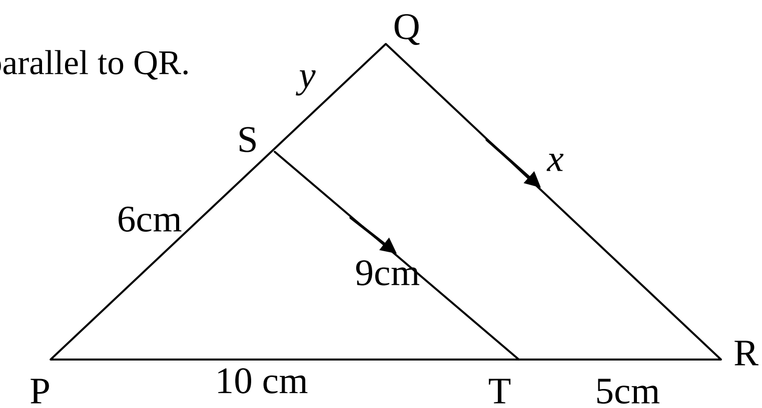
3. (a) Prove that the triangles shown below are similar.
   (b) Hence, work out the length of the sides labelled x and y.



4. After 2 years, a tree, S, 1.5m high has grown in height to tree, T.
The base of the trees, S and T are at a distance of 2.5m and 7.5m from a point, A, respectively.
Work out the height of tree, T. Give reasons.



5. In the diagram below ST is parallel to QR.



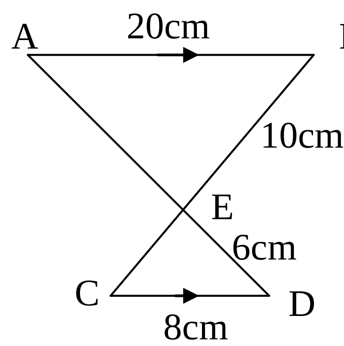
(a) Which two triangles are similar? Give reasons.
(b) Write down the ratio of the sides in the form 1: *n*.
(c) Hence, or otherwise, work out the value of *x* and the value of *y*.

*(d) What is the ratio of the area of triangle PST to the area of triangle PQR?

*(e) If the area of triangle PST is 48 cm^2, work out the area of the trapezium SQRT.

6. In the diagram below, AB is parallel to CD. AD and BC intersect at E.
AB = 20cm, CD = 8cm, BE = 10cm and DE = 6cm.
(a) Prove that triangles AEB and DEC are similar.
(b) Hence, work out the length of AE and CE.

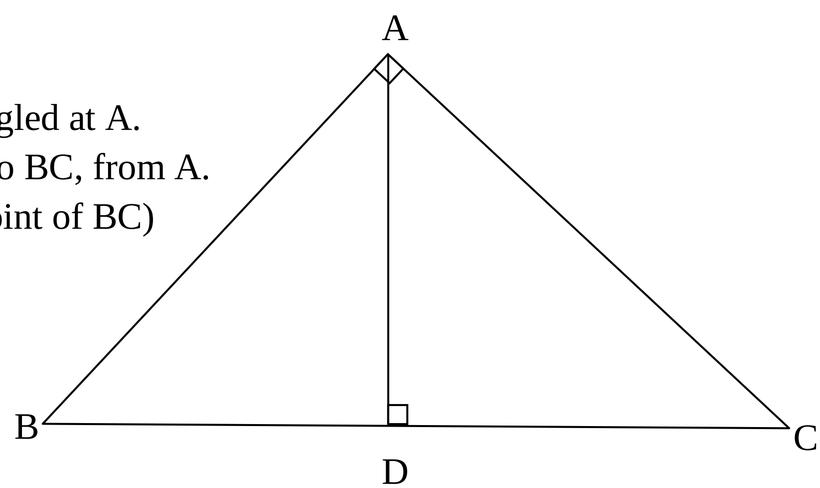
*(c) If the area of triangle DEC is 20cm^2, work out the area of triangle AEB.



KS4 HIGHER NOTE: ALL DIAGRAMS NOT DRAWN TO SCALE.

Challenging Questions:

7. Triangle ABC is right-angled at A.
BD is the perpendicular to BC, from A.
(Note: D is not the midpoint of BC)

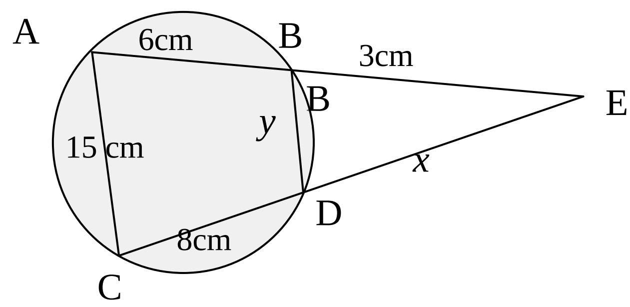


(a) Prove that triangles ABC and DBA are similar.
(b) Prove that triangles ABC and DCA are similar.
(c) Are triangles ADB and CDA similar? Give a reason.

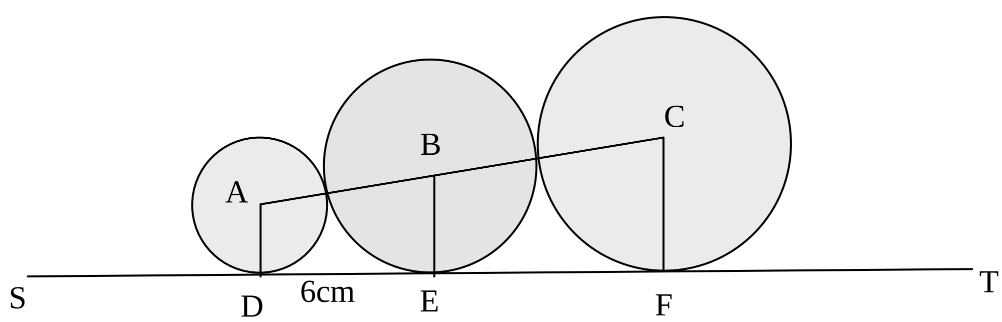
Given that BD = 9 cm and CD = 4 cm,

(d) Work out the length of AD.
(e) Work out the area of triangle ABC.
(f) Work out the length of AB, leaving your answer in the form $m\sqrt{13}$.

8. In the diagram shown, the two chords AB and CD of the circle are produced to intersect at E.
   (a) Using the exterior angle property of a cyclic quadrilateral or otherwise, prove that triangles AEC and DEB are similar.
   (b) Hence, work out the value of x and y.



9. Line ST is tangent to three circles of radii 3cm, 5cm and 7cm at the points D, E and F respectively. A, B and C are the centres of the circles.
   Given that DE = 6cm, work out the length of EF.



Answers/Solutions (solutions not unique).

1. A and B. Equiangular (AAA) angles are 90˚, 63˚ and 27˚ in each triangle.
   C and D. Equiangular (AAA) angles are 110˚, 40˚ and 30˚ in each triangle.
2. A and G. Ratio of sides = 5:15=3:9 =1:3.
   C and D. All squares are similar.
   B and E. Ratio of sides = 25:10=10:4=2.5:1
   I and H. All circles are similar.

NOTE: Polygons with more than 3 sides may be equiangular but the sides may not be in the same ratio. In such cases, they are not similar. For example, F and H are equiangular but not similar because $15{:}\,30 \neq 6{:}\,10$

It is not possible enlarge a rectangle 15cm by 6cm to a rectangle 30cm by 10cm.

3. (a) The third angle is 80˚ (sum of angles =180˚). Hence, the triangles are equiangular (AAA) and hence similar.
   (b) The ratio of the sides = 10:20 = **1:2**
   Hence, ***x* = 2x6 = 12 cm, and *y* = 18÷ 2 = 9cm**

4. The two right-angled triangles are equiangular (AAA), 90˚ in each, angle A is common to both and the third angle in each triangle must be equal by subtraction from 180˚. Hence, the triangles are similar
   and the ratio of the sides = 2.5 : 7.5 = 25: 75 = 1 : 3,
   Hence, **height of tree, H = 3 x 1.5 = 4.5m**

5. (a) Triangle PST and PQR.
   Reason: angle PST = PQR corresponding, angle PTS = PRQ corresponding
   Angle P is common to both triangles
   Hence, by AAA (equiangular) the triangles are similar.

   (b) Ratio of sides = 10:15 = 1: 1.5 (one and a half times)
   (c) Hence, ***x* = 1.5 x 9 = 13.5** (9 + 4.5 = 13.5)
   PQ = 1.5 x 6 = 9, hence, ***y* = 9 – 6 = 3.**

   (d) Ratio of lengths = 1:1.5, hence, ratio of areas = $1^2 : 1.5^2 = 1 : 2.25$

   (4: 9 is acceptable)

   (e) Area of triangle PQR= 2.25 x 48 = 48+48+12 = 108cm^2

   Note: 48 x 2.25 = 48(2 + 0.25) = 48x2 + 0.25 x 48 = 96+12=108

   Hence, area of SQRT = 108 – 48 = 60cm^2.

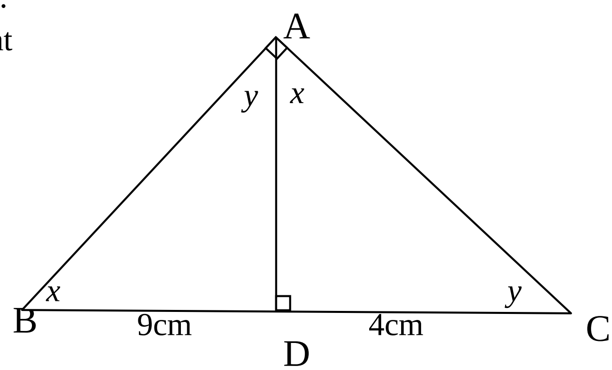
6. (a) Angle EAD = EDC alternate angles,
angle EBC = ECD alternate angles,
angle AEB = CED vertically opposite angles (or by subtraction from 180˚)
Hence, the triangles are equiangular (AAA) and hence similar.

(b) Ratio of sides = 8: 20 = 2:5 = **1: 2.5** (8:20 or 2:5 are also acceptable)

(c) Hence, **AE = 2.5 x 6 = 6 + 6 + 3 = 15cm.**

CE = 10 ÷ 2.5 = 4cm. Note: $\frac{10}{2.5}$ (double each) $= \frac{20}{5} = 4$

7. Let angle ABD = *x* and angle BAD = *y*.
From the triangle ABD, we can see that
$x + y = 90˚$.
But angle BAC = 90˚,
hence angle DAC = *x*.
Similarly, angle ACD = *y*.



(a) It is now easy to see that triangles ABC and DBC are equiangular (AAA) and hence similar.
(b) Similarly, triangles ABC and DCA are equiangular (AAA) and hence similar.
(c) Triangles ADC and CDA are equiangular (AAA) and hence similar.
(We could say that they are similar since they are both similar to ABC).
Please note that I have left it to you to write down details, stating which angles are equal.

(d) Since triangles ADB and CDA are similar, the ratio of the sides must be equal.
Hence, $\frac{AD}{CD} = \frac{BD}{AD}$ hence $AD^2 = CD \times BD = 4 \times 9 = 36$

Hence, **AD =** $\boldsymbol{\sqrt{36} = 6}$**cm**

(e) Area of triangle ABC = $\frac{1}{2} \times (9 + 4) \times 6 = \frac{1}{2} \times 13 \times 6$ = **39 cm^2.**
(f) Apply Pythagoras' Theorem to triangle ABD:

$\text{AB}^2 = 9^2 + 6^2 = 81 + 36 = 117,$
hence, $\text{AB} = \sqrt{117} = \sqrt{9 \times 13}$ = $\boldsymbol{3\sqrt{13}}$

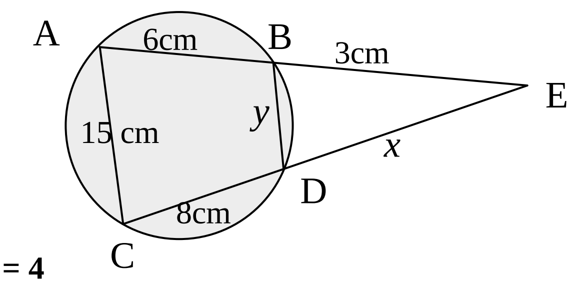
8. (a) **Triangle DEB** **Triangle AEC**

Angle E = angle E common angle

Angle EBD = angle ECA exterior angle of a cyclic quadrilateral is equal to the opposite interior angle

Angle EDB = angle CAE exterior angle property (as above)

The triangles are therefore equiangular (AAA) and hence similar.

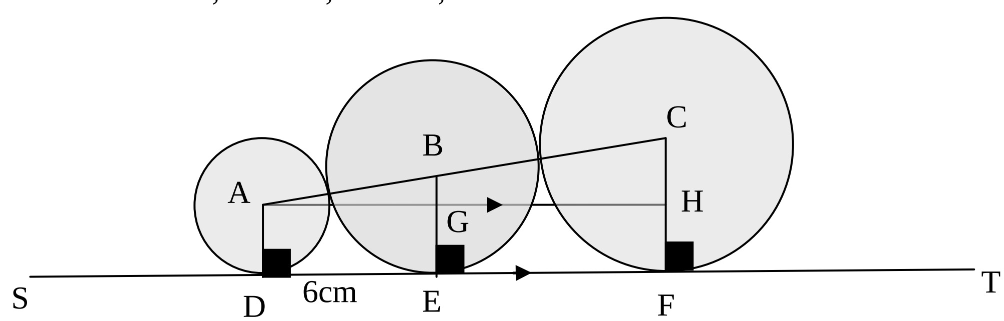


(b) The ratio of the sides = 3:9 = 1: 3.

Hence, ***y* = 15÷ 3 = 5**

Now, EC = x + 8 = 3x, Hence, 8 = 2x , ***x* = 4**

9.



Draw a line from the point A parallel to ST meeting BE at G and CF at H.

Consider the similar triangles BAG and CAH (reasons given below)

Angle BAG = CAH common, angle AGB = AHC = 90˚, angle GBA = HCA corresponding

Hence the triangles are equiangular (AAA) and therefore similar.

(Note also that a tangent and radius meet at 90˚)

The ratio of the lengths AB: AC = 8:20 = 2:5 = 1: 2.5

Hence, AH = 2.5AG = 2.5DE = 2.5x6 = 6+6+3 = 15cm. Hence **EF = 15- 6 = 9cm**

(Note: There is another way of doing this question).

I hope you find this useful. If you find any errors, please let me know.